



Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/gmcl19>

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Version of record first published: 24 Sep 2006

To cite this article: Sergei A. Podoshvedov & Fedor V. Podgornov (1998): Peculiarities of Energy Exchange Among Four Light Waves Spreading Forward in Media with Diagonal-Bipolar Response: Eigenmodes, Spatial Instability and Optical Switching, Molecular Crystals and Liquid Crystals Science and Technology. Section A. Molecular Crystals and Liquid Crystals, 321:1, 97-112

To link to this article: <http://dx.doi.org/10.1080/10587259808025079>

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Peculiarities of energy exchange among four light waves spreading forward in media with diagonal-bipolar response: eigenmodes, spatial instability and optical switching

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Interaction of four light waves in a nematic liquid crystal is studied theoretically. We graphically on the phase plane solve equations describing four wave mixing (FWM) under excitation of the static lattices of the dielectric permeability in the nematic liquid crystal. We show existence of unstable eigenmode in FWM. Effect of optical switching of energy is predicted. Energy exchange among the light waves depends on relative initial phase among the four waves. Experimental conditions, under which the effects that are studied would be observable, are discussed.

Keywords: four wave mixing, optical switching, eigenmodes, spatial instability

INTRODUCTION

Spatial instability is a general phenomenon that exists for a wide class of the wave processes in nonlinear optics. The existence of the spatial instability is connected with joint action of both the parametric energy exchange among the light waves and nonparametric interactions responsible for self and cross-phase modulation. For example, spatial instability has been discovered in nonlinear dynamics of the polarization of an intense beam in cubic crystals with birefringence and third-order nonlinearity^[1], the degenerate four-photon

mixing between orthogonal polarizations in a birefringent fiber^[2-4] and in the two-wave mixing of the fundamental and its third harmonics^[5]. Studying of given phenomenon is of special interest for improving of the design of the optical switching devices. But large laser intensity needed for obtaining of the effect of optical switching in media with third-order nonlinearity restrict their possibilities. In present paper we consider FWM in the nematic liquid crystal for which the dominant contribution in the dielectric permeability is of thermal origin. As will be shown below nematic liquid crystals can be used as candidates for further investigations and application in optical switching devices.

THEORY

It is well known that nematic liquid crystals have very dependence of the index of refraction on the temperature. For the mesophase of the nematic liquid crystals $\partial n_{\parallel}/\partial T$ is negative ($\approx -(4-6) \cdot 10^{-3}$ degree) while the value $\partial n_{\perp}/\partial T$ is positive but 4-5 times smaller. When own absorption of the nematic liquid crystal becomes more considerable then the nonlinearity connected with thermal effects determines the third-order susceptibility responsible for both energy exchange among the light waves and self and cross-phase modulation. Note that in the nematic liquid crystals, the most often, tensor of the dielectric permeability $\epsilon_{ij}^{(0)}$ has the following form:

$$\epsilon_{ij}^{(0)} = \epsilon_{\perp}^{(0)} + \epsilon_a^{(0)} n_i n_j \quad (1)$$

where $\epsilon_a^{(0)} = \epsilon_{\parallel} - \epsilon_{\perp}$ is anisotropy of the dielectric permeability; n_i is the component of the unit vector in a direction of the optical axis. Local change of the main components of tensor (1) in result of absorbing of the energy of the light waves can be presented as: $\delta \epsilon_{\perp, \parallel} \approx \partial \epsilon_{\perp, \parallel} / \partial T \cdot \delta T$, where δT is the

change of the temperature of the nematic liquid crystal. Evolution of the temperature field is governed by the equation of diffusion:

$$\frac{\partial \delta T}{\partial t} - \chi \Delta \delta T = \frac{\sigma c n |E|^2}{8\pi \rho C_p} - \Gamma \delta T \quad (2)$$

where Δ is laplacian; σ is the coefficient of absorption of the nematic liquid crystal (cm^{-1}); ρC_p ($\text{erg}/(\text{cm}^2 \cdot \text{degree})$) is thermal capacity per unit of volume $\chi \approx 10^{-3} (\text{cm}^2/\text{c})$ is the coefficient of the heat conductivity, and $|E|^2$ is intensity of the light waves. The second term in right part of Eq.(2) is introduced to take into account bend of the heat from the nematic liquid crystal under homogeneous heating;

$$\Gamma \approx \chi(a^{-2} + \pi^2/L^2)$$

is a inverse time of establishment of the static field of the temperature under applying of the external source of heat to the nematic liquid crystal, where a is transverse radius of the beam; L is thickness of the cell.

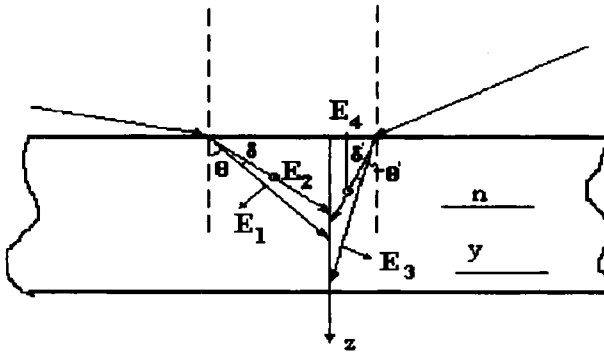


FIGURE 1 Geometry of interaction: four light waves propagate within the nematic liquid crystal. Beams E_1 and E_3 are tilted at the angles Θ and Θ' ; with helping of angles δ, δ' we take into account birefringence of the nematic liquid crystal.

Let us consider the case when four light waves degenerated on the frequency propagate in the nematic liquid crystal as shown in Fig.1 Two waves elliptically polarized fall to the nematic liquid crystal, whose optical axis is directed along \bar{y} . These waves fall apart on the four linearly polarized waves. The electromagnetic field can be presented as the superposition of the four monochromatic waves:

$$\bar{E} = \bar{e}_x(E_2(z)e^{i\bar{k}_2\bar{r}} + E_4(z)e^{i\bar{k}_4\bar{r}}) + \bar{e}_1E_1(z)e^{i\bar{k}_1\bar{r}} + \bar{e}_3E_3(z)e^{i\bar{k}_3\bar{r}} \quad (3)$$

where $\bar{e}_x, \bar{e}_y, \bar{e}_z$ are unit vectors of the Cartesian system; \bar{e}_1, \bar{e}_3 are unit vectors of polarization of the extraordinary light waves; E_2, E_4 and E_1, E_3 are electric-field amplitudes of the ordinary and extraordinary plane waves respectively. For further studying of FWM we suppose that both extraordinary and ordinary waves propagate along directions making small angle with axis \bar{z} . As consequence we neglect by influence of \bar{z} projections of the extraordinary waves on process of parametric energy exchange among the waves. We determine change of the dielectric permeability as: $\delta\epsilon_{xy} = \delta\epsilon_{yx} = 0$; $\delta\epsilon_{xx} = \delta\epsilon_{\perp}$; $\delta\epsilon_{yy} = \delta\epsilon_{\parallel}$. We neglect the changing of the dielectric permeability which is provoked by reorientation of the director. After substituting of Eq. (3) into Eq. (2) we can obtain an approximate solution:

$$\delta T = \frac{c}{8\pi\rho C_p} \Gamma^{-1} \Phi(\bar{r}; t) \quad (4)$$

where

$$\begin{aligned} \Phi(\bar{r}; t) = & (\sigma_e n_o (|E_2|^2 + |E_4|^2) + \sigma_e n_e (|E_1|^2 \cos^2 \Theta + |E_3|^2 \cos^2 \Theta')) (1 - e^{-\Gamma t}) + \\ & + \frac{\sigma_o n_o \Gamma}{\Gamma_o} E_2 E_4^* e^{i(\bar{k}_2 - \bar{k}_4)\bar{r}} (1 - e^{-\Gamma_o t}) + c.c. + \\ & + \frac{\sigma_e n_e \Gamma}{\Gamma_e} E_1 E_3^* \cos \Theta \cos \Theta' e^{i(\bar{k}_1 - \bar{k}_3)\bar{r}} (1 - e^{-\Gamma_e t}) + c.c. \end{aligned} \quad (5)$$

where $\Gamma_o = \chi(\bar{k}_2 - \bar{k}_4)^2$ and $\Gamma_e = \chi(\bar{k}_1 - \bar{k}_3)^2$. Eq.(5) is obtained for the case $\Gamma/\Gamma_e, \Gamma/\Gamma_o \ll 1$.

The equations describing parametric interaction of the four light waves under assumption $|\bar{k}_2 - \bar{k}_4|, |\bar{k}_1 - \bar{k}_3| \ll |\bar{k}_1|, |\bar{k}_2|, |\bar{k}_3|, |\bar{k}_4|$ have the following form^[8]:

$$\frac{dE_1}{dz} = -\frac{i\pi C_e}{\lambda n_e \cos^2 \Theta} ((\Theta_o + A_e b_e \cos^2 \Theta' |E_3|^2) E_1 \cos \Theta + A_o b_o \cos \Theta' E_2 E_3 E_4^* e^{i\Delta \bar{k}_z z}) \quad (6)$$

$$\frac{dE_2}{dz} = \frac{i\pi C_o}{\lambda n_o \cos(\Theta + \delta)} ((\Theta_o + A_o b_o |E_4|^2) E_2 + A_e b_e \cos \Theta \cos \Theta' E_1 E_3^* E_4 e^{-i\Delta \bar{k}_z z}) \quad (7)$$

$$\frac{dE_3}{dz} = -\frac{i\pi C_e}{\lambda n_e \cos^2 \Theta'} ((\Theta_o + A_e b_e \cos^2 \Theta |E_1|^2) E_3 \cos \Theta' + A_e b_e \cos \Theta E_1 E_2^* E_4 e^{-i\Delta \bar{k}_z z}) \quad (8)$$

$$\frac{dE_4}{dz} = \frac{i\pi C_o}{\lambda n_o \cos(\Theta' + \delta')} ((\Theta_o + A_o b_o |E_2|^2) E_4 + A_e b_e \cos \Theta \cos \Theta' E_1^* E_2 E_3 e^{i\Delta \bar{k}_z z}) \quad (9)$$

where

$$\Theta_o = A_o (|E_2|^2 + |E_4|^2) + A_e (|E_1|^2 \cos^2 \Theta + |E_3|^2 \cos^2 \Theta');$$

$$\Delta \bar{k}_z = (\bar{k}_2 + \bar{k}_3 - \bar{k}_1 - \bar{k}_4)_z$$

is projection of the wave mismatch in the direction of propagation (axis \bar{z});

λ is the wavelength;

$$C_o = \sigma c / 4\pi \rho C_p (dn_o/dT);$$

$$C_e = \sigma c / 4\pi \rho C_p \Gamma (dn_e/dT);$$

and $\sigma = (\sigma_o + \sigma_e)/2$ is average meaning of the coefficient of absorption; A_o, A_e, b_o, b_e are real coefficients; difference of A_o and A_e is determined by dichroism of absorption of the waves; the quantities b_o and b_e appoint effectiveness of exciting of the static lattices on comparing with homogeneous heating; $C_o > 0$, $C_e > 0$. It is worth noticing that $C_o = |C_o|$ and $C_e = |C_e|$ in Eqs. (6–9).

FWM in media with diagonal-bipolar nonlinearity was considered theoretically in Ref.^[8]. Analysis of the wave interaction was led in an approximation in which the depletion of the fundamental waves was not taken into consideration. The conditions realizing the effective energy exchange were found. It was shown that the amplification of the weak waves takes place in the definite range of the pump intensity. Aim of the present work is to study some peculiarities of the FWM. In particular, as will be shown, the output distribution of the power among the waves is very sensitive to the change of the initial conditions both the input power ratio and the relative phase. This effects can be used for the construction of optical switching devices. Eqs. (6–9) with boundary conditions are the basis for further consideration. The following correlations:

$$a |E_1|^2 + |E_2|^2 + b |E_3|^2 + c |E_4|^2 = P \quad (10)$$

$$a |E_1|^2 - |E_2|^2 = D_1 \quad (11)$$

$$b |E_3|^2 - c |E_4|^2 = D_2 \quad (12)$$

can be obtained from Eqs. (6–9), where

$$a = (A_e b_e C_o n_e \cos^3 \Theta) / (A_o b_o C_e n_o \cos(\Theta + \delta));$$

$$b = (A_e b_e C_o n_e \cos^3 \Theta') / (A_o b_o C_e n_o \cos(\Theta + \delta));$$

$$c = \cos(\Theta' + \delta') / \cos(\Theta + \delta).$$

P can be interpreted as conservation of the full power with taking into account the dispersion properties of the medium. The quantities D_1 , D_2 show that energy exchange occur only among the waves orthogonal polarized, in particular, between the ordinary and extraordinary light waves. This remarkable property distinguishes interaction of the light waves in medium with diagonal-bipolar nonlinearity from, for example, propagation of the light waves in materials with inversion symmetry (cubic media) where the lowest-order nonlinear effects originate from the usual third-order susceptibility $\chi^{(3)}$. Let us introduce the new variables q_1 , q_2 , q_3 and q_4 , so that:

$$E_1 = \sqrt{\frac{P}{a}} q_1 \quad (13)$$

$$E_2 = \sqrt{P} q_2 \quad (14)$$

$$E_3 = \sqrt{\frac{P}{b}} q_3 \quad (15)$$

$$E_4 = \sqrt{\frac{P}{c}} q_4 \quad (16)$$

Then Eqs. (6–9) can be transformed to a system of two equations for real quantities:

$$\eta(s) = |q_2|^2$$

and

$$\psi = ks + \varphi_2 + \varphi_3 - \varphi_1 - \varphi_4$$

where φ_i ($i = 1-4$) is the phase of the corresponding wave. That system has the following form:

$$\frac{d\eta}{ds} = \sin \psi \sqrt{\eta(d_1 + \eta)(1 - d_1 + d_2 - 2\eta)(1 - d_1 - d_2 - 2\eta)} \quad (17)$$

$$\begin{aligned}
\frac{d\psi}{ds} = & \cos\psi \left(\frac{\sqrt{(d_1+\eta)(1-d_1+d_2-2\eta)(1-d_1-d_2-2\eta)}}{2\sqrt{\eta}} - \right. \\
& - \frac{\sqrt{\eta(d_1+\eta)(1-d_1-d_2-2\eta)}}{\sqrt{1-d_1+d_2-2\eta}} - \frac{\sqrt{\eta(d_1+\eta)(1-d_1+d_2-2\eta)}}{\sqrt{1-d_1-d_2-2\eta}} + \\
& + \left. \frac{\sqrt{\eta(1-d_1+d_2-2\eta)(1-d_1-d_2-2\eta)}}{2\sqrt{d_1+\eta}} \right) - \eta(r_1+r_2+r_3+r_4) + \\
& + r_1 \frac{1-d_1-d_2}{2} + r_2 \frac{1-d_1+d_2}{2} - r_4 d_1 + k
\end{aligned} \quad (18)$$

where $s = z\rho$; $k = \Delta \bar{k}_z / \rho$;

$$\rho = (\pi C_e A_o b_o P) / (\lambda n_e) \sqrt{(\cos(\Theta + \delta) \cos \Theta) / (\cos(\Theta' + \delta') \cos \Theta')};$$

$$d_1 = D_1 / P; \quad d_2 = D_2 / P;$$

coefficients r_i ($i = 1-4$) depend only on the parameters of medium and the geometry of experiment; obtaining r_i we supposed that $\cos \Theta = \cos \Theta'$ and $\cos(\Theta + \delta) = \cos(\Theta' + \delta')$;

$$r_1 = r_3 = 1/r_2 = 1/r_4 = (C_o n_e \cos \Theta) / (C_e n_o \cos(\Theta + \delta)).$$

The system of Eqs. (6-9) can be represented in a Hamiltonian form for the two canonical-conjugate variables η and ψ :

$$\frac{d\eta}{ds} = - \frac{\partial H}{\partial \psi} \quad (19)$$

$$\frac{d\psi}{ds} = \frac{\partial H}{\partial \eta} \quad (20)$$

$$\begin{aligned}
H = & \cos\psi \sqrt{\eta(d_1+\eta)(1-d_1+d_2-2\eta)(1-d_1-d_2-2\eta)} - \\
& - \frac{\eta^2}{2} (r_1 + r_2 + r_3 + r_4) + \eta \left(r_1 \frac{1-d_1-d_2}{2} + r_2 \frac{1-d_1+d_2}{2} - r_4 d_1 + k \right)
\end{aligned} \quad (21)$$

$|q_1|^2, |q_3|^2, |q_4|^2$ can be found with helping d_1 and d_2 as:

$$|q_1|^2 = \eta + d_1;$$

$$|q_3|^2 = (1 - d_1 + d_2 - 2\eta)/2;$$

$$|q_4|^2 = (1 - d_1 - d_2 - 2\eta)/2.$$

This new representation gives the possibility to understand the nature of the dynamics of the FWM in the nematic liquid crystal. With helping Eq. (21) we can obtain both analytical and geometrical solution of the Eqs. (6–9). But exact analytical solution involves Jacobian elliptic functions. These expressions do not give an immediate insight into the physics of the interaction of the four light waves in the nematic liquid crystal. For simplicity, we do not present analytical solutions of Eqs (6–9). Information about the process of the FWM will be obtained by means of geometrical description which gives real advantages in comparison with the analytical solution.

RESULTS

In further consideration we consider the case $d_1 = d_2 = 0.1$. Meanings of the values d_1 and d_2 are changed from -1 up to 1 by changing input power ratio among the light waves. Let us solve Eq. (21) in the diagram form ($H = \text{const}$). The topology of these trajectories is determined by the number, location in the phase plane and spatial stability of the nonlinear eigenmodes (η_e, ψ_e) . These eigenmodes are extremal points of H

$$(\partial H / \partial \eta|_{(\eta_e, \psi_e)} = \partial H / \partial \psi|_{(\eta_e, \psi_e)} = 0).$$

Fig. 2 shows the fraction of the power in the eigenmode $\eta_e = |E_2|^2 / P$ ($r = r_1 + r_2 + r_3 + r_4 = 8.5$) as function of the dimensionless mismatch k . The spatially stable eigenmodes are represented by solid curves while the dashed

curves correspond to the unstable eigenmode. As shown in Fig. 2, there are two eigenmodes with $\psi_e = 0$ and $\psi_e = \pi$. A stability analysis shows that the unstable eigenmode with $\psi_e = \pi$ for given values d_1 and d_2 arises whenever $r > r_{cr} = 4.33$. Existence of the unstable eigenmode in the interaction of the four light waves in the nematic liquid crystal is connected with essential distinction of the coefficients of coupling C_o and C_e for the ordinary and extraordinary waves in Eqs. (6–9).

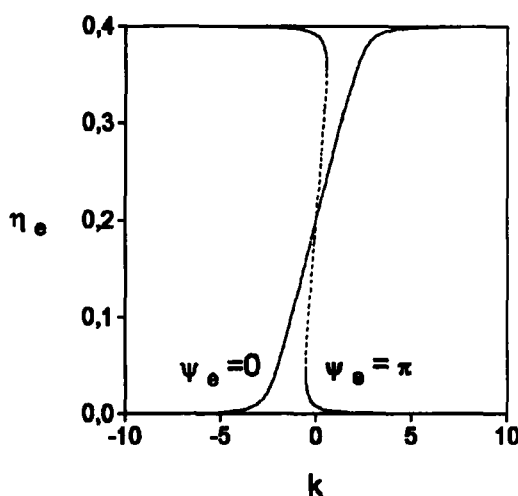


FIGURE 2 Bifurcation diagram: η_e is the eigenmode fraction of the power in the wave E_2 ; k the normalized mismatch. The stable (unstable) eigenmodes are shown by the solid (dashed) lines.

In general, the conversion efficiency of the FWM in the nematic liquid crystal for any initial choice of the initial phase difference, amplitude ratio and total intensity of the four waves can be graphically displayed by phase-plane portraits as in Fig.3(a,b). These figures show the level curves with

$H = \text{const}$ that are traced by the point (η, ψ) for the two different values of the linear mismatch k . Figures 3(a) and 3(b) have been obtained for $k = 0$ and $k = 0.4$ respectively. The circle of the radius $\eta = 0.4$ is the phase space of Eq. (21). As can be seen from Fig. 3(a,b) the stable eigenmodes are the stable centers. The unstable eigenmode with $\psi_e = \pi$ is an unstable saddle. This unstable eigenmode (it is indicated by the letter A in Fig. 3(a,b)) is the origin of a double-loop separatrix (it is singled out by thick lines).

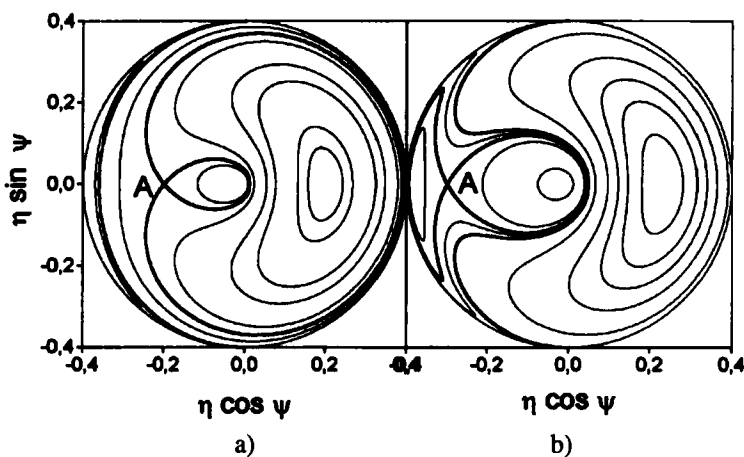


FIGURE 3 Phase-plane portraits for a) $k = 0$; b) $k = 0.4$

Besides the double-loop separatrix there is simple separatrix that emanates from the circle at the point with $(\eta = 0.4, \psi = 3\pi/2)$ and return back to the circle at the point with $(\eta = 0.4, \psi = \pi/2)$. These separatrices separate four qualitatively different types of periodic solutions of Eqs. (6–9). As can be seen from Fig. 3(a,b) the phase-plane portraits have essential distinctions. For example, large loop of the double-loop separatrix for the case $k = 0.4$ is restricted by area where $\cos \psi < 1$ on the phase-plane unlike the case $k = 0$.

Simple separatrix passes near the point $\eta = 0$ for $k = 0$ unlike the case $k = 0$.

It is instructive to demonstrate by two examples the physical insight that can be gained in the behavior of the frequency-conversion process by using the phase-plane portraits as in Fig. 3(a,b). Now we consider influence of the initial difference of the phases of the waves $\psi_0 = \psi(s=0)$ on amplification of the weak waves for the case $k = 0$.

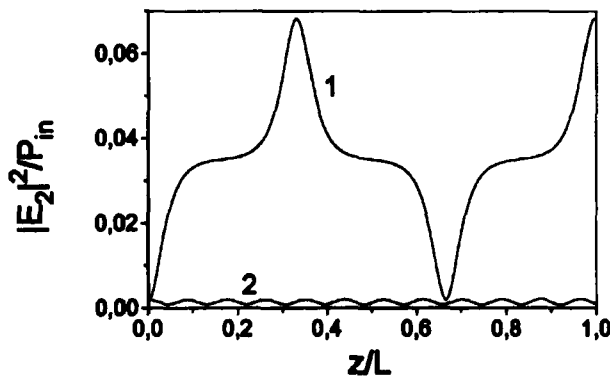


FIGURE 4 $|E_2(s)|^2/P_{in}$ as function of $s = z/L$. Curves 1 and 2 have been obtained for $\psi_0 = \psi(s=0) = 0$ and $\psi_0(s=0) = \pi$ correspondingly.

Fig. 4 shows the change of the fraction of the total input power in the extraordinary wave E_2 ($|E_2(s)|^2/P_{in}$ where

$$P_{in} = |E_{10}|^2 + |E_{20}|^2 + |E_{30}|^2 + |E_{40}|^2; \quad E_{i0} = E_i(z=0); \quad P_{in}(s) \neq const$$

along propagation through the nematic liquid crystal.

The curves 1 and 2 have been obtained for $\psi_0 = 0$ and $\psi_0 = \pi$ respectively.

Initial distribution of the power are the following: $|E_{10}|^2/P_{in} = 0.1728$;

$$|E_{20}|^2/P_{in} = 2.023 \cdot 10^{-3}; |E_{30}|^2/P_{in} = 0.757; |E_{40}|^2/P_{in} = 0.06815; \quad \rho L = 50;$$

L is the length of the crystal. As shown in Fig. 4 amplitude of oscillation of the $|E_2(s)|^2/P_{in}$ can be abruptly increased in the case $\psi_0 = 0$ unlike in the case $\psi_0 = \pi$. There is energy conversion from light waves E_3 and E_4 to the waves E_1 and E_2 . Fractions of the total initial power in ordinary (E_2) and extraordinary (E_1) light waves can grow approximately in 33.4 and 4.4 times correspondingly under definite length of the nematic liquid crystal and total initial power. This curious behavior could have been easily anticipated by inspecting of the fate of the points with equal meanings of the value $\eta_0 = \eta(s=0)$ but with different ψ_0 .

The phase portraits in Fig. 3(a,b) can be used for studying of the phenomenon of the switching of power too. Both spatial frequency of the conversion among the waves and amplitude of the oscillation feel small changes in the initial distribution of the power among the waves near the double-loop separatrix. Let us consider the case $k=0$ and the distribution of the power among the waves are the following: $|E_{10}|^2/P_{in} = 0.7579$; $|E_{20}|^2/P_{in} = 0.0682$ $|E_{30}|^2/P_{in} = 0.174$; $|E_{40}|^2/P_{in} = 0.002$; $\psi_0 = 0$; $\rho L = 50$. In Fig. 5 we show the dependence of the fraction of the total initial power of the ordinary wave E_2 as function of the dimensionless propagation distance z/L .

When the initial point with $\psi_0 = 0$ is outside of the double-loop separatrix then the point will evolve as curve 2 in Fig. 5. In this case the conversion of the wave E_2 is relatively small. If the fraction of input total power in the wave E_2 is such as for the curve 2 minus weak in-phase seed so that the initial point crosses the separatrix then the evolution of this point is rather different from the curve 2.

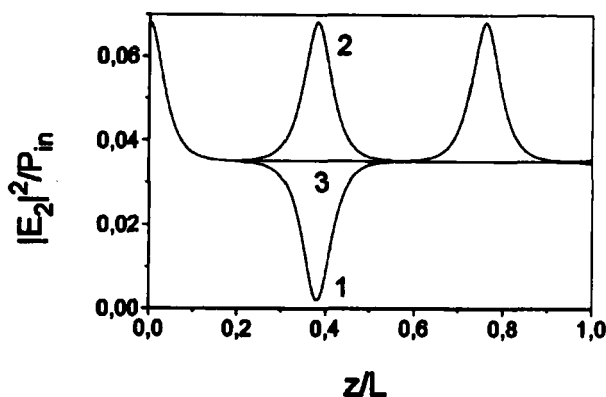


FIGURE 5. Doubling of the spatial period of the frequency conversion ($|E_2(s)|^2/P_{in}$, versus $s = z/L$), induced by a slight change of the input power fraction in the wave E_2 . Curves 1, 2, 3 have been obtained for the cases when the initial points are inside (1), outside (2), and on the separatrix (3).

Curve 1 has been obtained for the case when initial distribution of the power among the waves corresponds to the point lying on the left of large loop of the separatrix. Initial distinction of the meaning $|E_2(s=0)|^2/P_{in}$ for the curves 1 and 2 is no larger 10^{-6} . Spatial frequency of the conversion among the waves for the curve 1 is doubled. When the curve 2 has a maximum then the curve 1 has minimum under the definite length of medium. This effect may be important in creating and improving the design of all optical switching devices based on the thermal nonlinearity of the nematic liquid crystal. The ratio between the maximum of the curve 2 and the minimum of the curve 1 is 33.4 for ordinary and 4.3 for extraordinary waves. For curve 3, the point on the phase plane moves on the separatrix towards the unstable eigenmode (i.e. point A).

Now we are going to discuss the experimental conditions under which the effects that are studied would be observable. Let us consider the propagation of the light waves with the wavelength $\lambda = 0.5\mu\text{m}$. All our calculations will be carried out with the following meanings: $a = 0.1\text{cm}$; $L = 100\mu\text{m}$; $\rho C_p \approx 1.5 \cdot 10^7 \text{ erg/cm}^3 \cdot \text{degree}$; $n_e = 1.71$; $n_o = 1.51$; $\sigma = 5\text{cm}^{-1}$; $A_o \approx A_e = 1$; $b_o \approx b_e \approx 0.5$. Using these values we can obtain $\Gamma = 10^3 \text{ c}^{-1}$. Our estimations show that the needed laser intensity under which both considerable increasing of the power under changing of the initial relative phase ψ_0 , and optical switching of the power would be observed is $\approx 7.7 \cdot 10^4 \text{ W/cm}^2$ and $\approx 8.8 \cdot 10^4 \text{ W/cm}^2$ respectively. The time of establishment of the studied nonlinearity is much more than that in the cubic crystal. But needed intensity is smaller in 10^{-7} times of the intensity which is required for observing of optical switching in the cubic crystal.

CONCLUSION

The nonlinear dynamics of the FWM on the thermal nonlinearity of the nematic liquid crystal was studied by means of a geometrical description. Parametric instability was observed. As shown in this work, we can obtain switching of the power from two waves to other by means of changing both the input distribution of the power among waves and initial ψ_0 under the definite length of the nematic liquid crystal and the total power. The length of the nematic liquid crystal and the total intensity can be used as adjusting parameters. It is worth hoping that results our work can be used for creating of optical switching on the nematic liquid crystals.

Acknowledgments

The authors would like to thank Dr. M. Kreuzer, the head of the liquid crystal group at Institute of Applied Physics, Darmstadt, Germany for his help and consideration, as well as, German Science Foundation for its financial support.

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